

Neutron Stars Properties and Crust Movements in Post-glitch Epoch

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Using a new numerical code with non-uniform adapted mesh, we study the changes produced in the global properties of neutron stars by the motion of matter in crust region during post-glitch epoch. Our numerical analysis shows that these changes may contribute to explain the observed spin-down of rotational frequency.

PACS numbers: 97.60.Gb, 97.60.Jd

The sudden spin jumps in rotational frequencies or “glitches” observed in many pulsars [1], as well as the transient phase “post-glitch” returning to the continuous spin-down process, are the main features of the pulsar evolution and provide great information sources to study the structure and evolution of neutron stars [2].

The core of neutron stars is mainly composed of superfluid neutrons, protons, and relativistic degenerate electrons (protons and electrons are found in a small fraction of the neutron abundance). In the outer kilometer protons are trapped in a lattice of neutron-rich nuclei (the “crust”). The region between nuclei is filled by neutron superfluid [3].

The crust and the core of the neutron star are strongly coupled and rotate with an angular velocity Ω_c . On the other hand, neutron superfluid, in order to rotate with an angular velocity Ω_s , establishes an array of quantized vortex lines parallel to the neutron spin axis (these vortex lines are surrounded by a standard matter core) [4, 5]. It is thought that “glitches” represent a variable coupling between the core crust component and the neutron superfluid [4].

The continuous spin-down due to the angular momentum loss, via electromagnetic radiation caused by the magnetic torque, produces stresses in the crust that eventually can be broken and brought to its more spherical equilibrium configuration. The decrease of the moment of inertia produces a sudden spin jump, which may explain small glitches [6] but not the large (giant) ones observed in Vela and other pulsars.

Several models use the superfluid as a reservoir of angular momentum by the mechanism of vortex pinning and unpinning. Vortices can pin to the nuclei lattice in the inner crust [7], not allowing for a radial component in their velocity. As a consequence, the superfluid stores angular momentum [5]. A sudden unpinning of vortices and their subsequent motion in the outer direction produce a rapid spin-down in the superfluid. As the angular momentum can be considered constant, this also produces a sudden spin-up of the core-crust component of the star, which gives rise to the observed glitch.

Different models have been proposed in order to explain the glitch phenomena: mechanical glitches [8], thermal glitches [9], core driven glitches [10], glitches based on centrifugal buoyancy forces [11], catastrophic unpinning [12], or annihilation of proton flux tubes in the crust core boundary [13]. In some of these models we found a motion of matter away from the rotation axis.

In some models the post-glitch transitory epoch is thought to correspond to a vortex repinning period. In order to explain the exponential and linear behavior found from fits of experimental data of pulsar frequency, Alpar et al (1984, 1993) have considered the existence of different regions in the crust with a different behavior in the unpinning process. They differentiate the regions with vortex motion in the glitch and those without vortex motion. Also, the phonon-vortex interaction has been considered to explain the post-glitch spin-down [15]. Another model is the crust-cracking model developed by Franco et al (2000), with the motion of matter to higher latitudes in the crust induced by starquakes. Recently, other model for the post-glitch epoch, based on a model for glitches by Ruderman et al (1998), has been proposed [17].

In this paper, we will study the evolution of a neutron star in the post-glitch epoch. In order to do that we will obtain a family of hydrostatic equilibrium configurations by solving numerically the partial differential equations and the corresponding boundary conditions for different times in this epoch. We will use a mesh adapted to the boundary of the star to avoid the high frequency oscillations (Gibbs phenomenon) observed near the discontinuity surfaces (in our case the star boundary) in codes with non-adapted meshes [18].

We assume that, just after the crust cracking, the neutron star reaches a quasi-equilibrium configuration with a shape corresponding to the angular velocity at the glitch epoch (the adapting process is almost instantaneous, as experimental observations indicate). From this moment and during the post-glitch (which may take from weeks to months, the time needed to achieve a situation allowing for the formation of a new solid crust), the neutron star

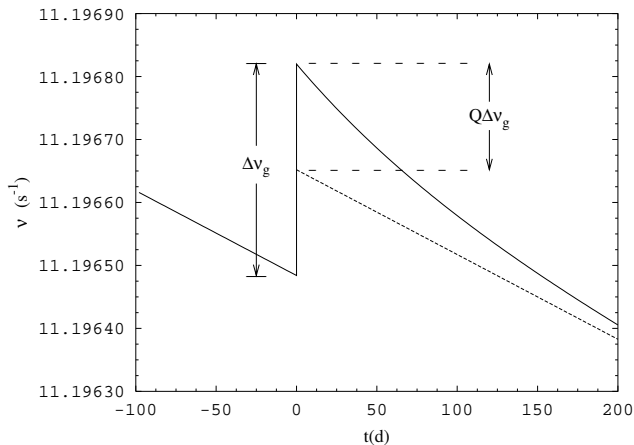


FIG. 1: Frequency evolution in a typical large glitch in a pulsar of the Vela type.

takes its quasi-equilibrium shape, in almost an instantaneous manner, following the evolution of the rotation frequency and without the constraining stresses of the crust, which now is broken.

Note that in the problem there are two different characteristic times (the rotation period and the duration of the post-glitch epoch).

Our model of neutron star is composed by two regions; The core and the crust region. The star boundary (a surface of constant gravitational plus centrifugal potential) is completely free (in general, it does not correspond to an ellipsoid) and it changes with time. We will make the following approximations: the core can be described by a constant density perfect fluid (given the results for the density profile obtained in the studies that use physical equations of state this seems to be a good first approximation), the crust region can be described by a surface density shell corresponding to a jump in the normal derivative of the gravitational potential on the star's boundary (the crust is a thin layer in the outer part of the neutron star), and the angular velocities of the core and the the crust are equal. Also, we impose axial and equatorial symmetry in our Newtonian study (a general relativistic version is under development).

We assume that in the post-glitch epoch there is a motion of matter in the crust region; in our model this corresponds to changes in the matter density in the shell found in the boundary of the star which represents this region. The stratification expected in the crust region [8, 17] supports this hypothesis. This matter motion may represent a matter motion related with vortex motion in the repinning process, which is favored near the poles of the star [5], or crust plates motion to the magnetic poles [16], or magnetized patches motion [8]. In our study we determine a matter motion in the crust region compatible, inside our framework, with the frequency evolution observed (Fig. 1). This motion of matter in the crust region produces changes in the global properties of the

neutron star. We will analyze this effect and its influence on glitches description.

For each time, in the post-glitch epoch, we will obtain a matter distribution in the crust region as well as the corresponding neutron star hydrostatic equilibrium configuration. The time evolution of the matter distribution in the crust region and the rest of the properties of the neutron star, are determined by fitting the rotation frequency and the angular momentum of the calculated configurations to the values observed and predicted, respectively, for these quantities during the post-glitch epoch.

Any hydrostatic equilibrium configuration of the family is obtained by solving the Euler and Poisson equations in the interior of the neutron star and the Laplace equation in the exterior region. The star boundary is determined by the implicit equation $p(r, \theta) = 0$, where $p(r, \theta)$ is the pressure; note that the boundary has to be found at the same time that the partial differential equations are solved (i.e., it is a free boundary problem). We have to impose the following boundary conditions on the gravitational potential U ; $U_{in|S} = U_{out|S}$ and $\partial_n(U_{in} - U_{out})|_S = \sigma(\theta)$, where $\sigma(\theta)$ describes the surface density profile in the crust region and ∂_n means normal derivative to the surface S .

To solve this problem we have developed a numerical code based on the program CADSOL [19] (a program used for the numerical solution of elliptic partial differential equations by Newton-Raphson methods, for instance, it has been successfully used to study Einstein-Yang-Mills fields [20]). Our code uses an iterative process to calculate the surface of the star and the solutions of the equations on a boundary adapted non-uniform mesh. In each iteration the star boundary is moved and the mesh re-adapted to the new boundary; the Poisson and Laplace equations are solved with the new boundary conditions and then the Euler equation determines a new boundary. The process is repeated up to the moment that all the equations are verified with a given precision (10^{-8}).

Now, let us consider the spin frequency time dependence in a typical glitch, Fig. 1. The post-glitch epoch is fitted using an exponential dependence on top of a linear dependence [4] (sometimes better fits are obtained with several exponential terms; here we use a model with just one exponential term for simplicity). Then the post-glitch frequency can be described by the following expression

$$\nu(t) = \nu_0(t) + \Delta\nu_0[Qe^{-t/\tau} + 1 - Q], \quad (1)$$

where $\nu_0(t) = at + b$. The meaning of the parameters appears in Fig. 1. The actual values of the parameters are fixed by the best fit of the experimental timing data. Here we use a simulation with the following parameters

$$Q = 0.5, \quad \tau = 100 \text{ days}, \quad \Delta\nu_0 = 3.35894519466 \cdot 10^{-4}, \\ a = -1.34623296 \cdot 10^{-6}, \quad b = 11.1964839822. \quad (2)$$

These parameters are of the order of magnitude of those obtained in a real large glitch. Note that the spin jump

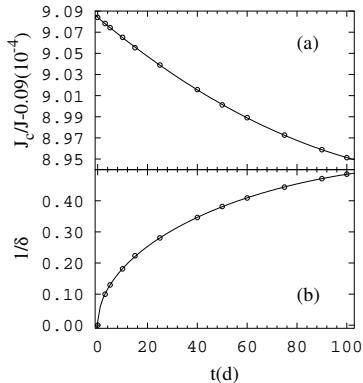


FIG. 2: Crust-superfluid density parameter and angular momentum transfer evolution.

value is slightly over a typical one for a large glitch. This is due to the fact that even when the precision obtained in our numerical calculations is excellent, we are in the threshold to fit the exponential data of a real glitch. Some changes in our routines are under development to improve the precision and avoid this limitation.

We concentrate our attention on the one hundred days just after the glitch. First, we have to calculate the configuration at the glitch epoch ($t = 0$) after the spin jump. This configuration is characterized by the spin frequency, core density, crust-superfluid density (for $t = 0$, in a conservative manner, we use uniform density), and core and crust-superfluid mass. The shape, angular momentum, and the rest of the physical properties of the neutron star are then obtained.

The continuous loss of angular momentum, due to the electromagnetic torque, has also to be considered in the post-glitch epoch. Here, we assume, in a simplified form, that this angular momentum loss is proportional to the linear part of the spin frequency fit (implicitly, we are assuming that in average the linear changes in the angular velocity of the neutron star is due, in first approximation, to the electromagnetic radiation; in general, other effects can contribute to this linear part [5]). The constant of proportionality is obtained from the angular momentum we calculate for the configuration at the glitch epoch after the spin jump.

Now, in the post-glitch epoch, we have the spin frequency and the estimated angular momentum of the neutron star for each time t . In order to generate our model we must indicate the form of the crust-superfluid region surface mass density. The effect we want to show is not very sensitive to the particular choice of the density profile (our model allows a completely general form for the density profile). We choose a profile for the crust-superfluid density consisting of a uniform term plus a Gaussian distribution centered around a latitude near the

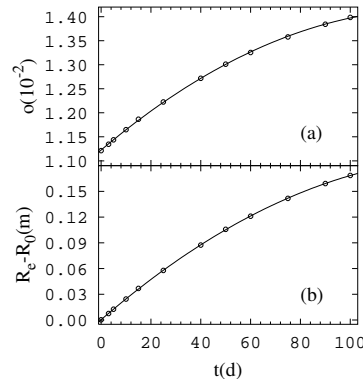


FIG. 3: Oblaticity and equatorial radius evolution.

poles ($\pi/12$ in our case), with relative weights 1 : 0.01,

$$\sigma = K \left[1 + \frac{0.01 e^{-\frac{(\theta - \pi/12)^2}{2\delta^2}}}{\int_0^{\pi/2} e^{-\frac{(t - \pi/12)^2}{2\delta^2}} dt} \right]. \quad (3)$$

Here K and δ do not depend on θ . (Remark: Concrete values 0.01 and $\pi/12$ have no special influence on the results described below). We also assume constant mass of the neutron star, constant core density, and constant ratio between the crust-superfluid mass M_s and the core mass M_c , $\frac{M_s}{M_c} = 0.06$. Then the observed spin frequency and the predicted angular momentum of the configuration determine the shape of the star and the actual crust-superfluid region density profile.

In order to visualize the physical properties for the configurations in the family we use two parameters δ and $o = \sqrt{1 - (R_p/R_e)^2}$ (oblativity). Note that this parameter o will not characterize completely the shape of the star, because it is not spheroidal.

To obtain the values of these two parameters for each time t we calculate curves of constant angular momentum in the two-dimensional parameter space $o - \delta$. In order to do that, we choose a value for δ and then we move the oblativity to compute an hydrostatic equilibrium configuration with the angular momentum of the neutron star at the time t . The calculated angular velocity for this configuration will not be in general the actual one of the neutron star at this time. Owing to that, we have to change the value of δ , and repeating the previous algorithm obtain another configuration. All the process continues by interpolation method up to the moment the calculated frequency and angular momentum coincide with those of the neutron star at this time t , within a relative error below 10^{-7} . We must repeat this iterative scheme for several values of t during this post-glitch epoch.

The results for a neutron star with total mass $M = 1.4M_\odot$ and initial equatorial radius $R_0 = 12$ km are presented in Figs. 2 and 3. In Fig. 2.b we plot the evolution

of the reciprocal of the density parameter δ . We observe that the pulsar spin-down, in the post-glitch epoch, represents an increase of $1/\delta$, which means an increase of the crust-superfluid density near the poles (matter approaching the axis). In principle, this sounds contradictory because a motion of matter to the axis should decrease the moment of inertia and then a spin-up is expected. The paradox is solved if we consider the reaction of the neutron star to this motion; if we look at the results presented in Figs 3.a and 3.b, we observe that the star reacts increasing its oblateness and equatorial radius, i.e., moving mass away from the axis. This fact compensates the motion of matter to the axis in the crust region, which gives rise to the final result of an increase of the total moment of inertia and the spin-down of the pulsar. The transfer of angular momentum from the crust region J_c to the core is presented in Fig. 2.a. The effect on the frequency of this transfer is overbalanced by the increased moment of inertia.

We have now a new and unexpected effect; the motion of matter to the axis in the crust region (due to crust plates motion, vortex repinning or any other origin) produces a reduction in the pulsar frequency due to the neutron star reaction. In general, this effect will collaborate with the rest of effects proposed in the literature to give the final spin-down observed.

Also, we would like to mention that the same effect has been observed in a constant angular momentum family of configurations, assuming a motion of matter to the equa-

tor in the crust-superfluid shell. Applying this result to the spin jump in the glitch epoch, which can be considered as a constant angular momentum phenomenon, we observe that the neutron star reacts decreasing its oblateness and approaching mass to the axis. Then the total moment of inertia is reduced and the result is a spin up of the neutron star.

Our model have some limitations such as, constant density for the core and the Newtonian framework. However, we presents for the first time a global study of neutron star behaviour in the post-glitch epoch which takes into account the crust region. New numerical codes are under development to improve the model and we are very confident that this new effect will remain in the scene, collaborating with the rest of known phenomena to describe glitches, allowing us to have a more complete picture of pulsar's evolution.

We wish to thank J. Kunz and B. Kleihaus for getting us started on CADSOL.

Acknowledgments

L.M.G-R. is supported by Spanish Ministry of Education and Science Project FIS2005-05198. F.N-L. is supported by Spanish Ministry of Education and Science Project FIS2006-12783-C03-02

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- [1] V. Radhakrishnan, and R. N. Manchester, *Nature*, **222**, 228 (1969); P. E. Reichley, and G. S. Downs, *Nature* **222**, 229 (1969); P. E. Boynton, E. J. Groth, D. P. Hutchinson, G. P. Nanos, R. B. Partridge, and D. T. Wilkinson, *Astrophys. J.* **175**, 217 (1972); J. M. Cordes, G. S. Downs, and J. Krause-Polstorff, *Astrophys. J.* **330**, 847 (1988); C. S. Flanagan, *Nature* **345**, 416 (1990); N. Wang, R. N. Manchester, R. T. Pace, M. Bailes, V. M. Kaspi, B. W. Stappers, and A. G. Lyne, *Mon. Not. R. Astron. Soc.* **317**, 843 (2000); T. Wong, D. C. Backer, and A. G. Lyne, *Astrophys. J.* **548**, 447 (2001).
 - [2] C. P. Lorenz, D. G. Ravenhall, and C. J. Pethick, *Phys. Rev. Lett.* **70**, 379 (1993); B. Link, R. I. Epstein, and J. M. Lattimer, *Phys. Rev. Lett.* **83**, 3362, (1999).
 - [3] J. M. Lattimer and M. Prakash, *Phys. Rep.* **333**, 121 (2000); H. Heiselberg and V. R. Pandharipande, *Ann. Rev. Nucl. Part. Sci.* **50**, 481 (2000); N. K. Glendenning, *Compact Stars: nuclear physics, particle physics, and general relativity*, Astronomy and Astrophysics library, Springer-Verlag, New York (1996).
 - [4] G. Baym, C. J. Pethick, D. Pines, and M. Ruderman, *Nature* **224**, 872 (1969).
 - [5] M. A. Alpar, P. W. Anderson, D. Pines, and J. Shaham, *Astrophys. J.* **276**, 325 (1984).
 - [6] M. Ruderman, *Nature* **223**, 597 (1969).
 - [7] P. W. Anderson and N. Itoh, *Nature* **256**, 25 (1975).
 - [8] M. Ruderman, *Astrophys. J.* **366**, 261 (1991).
 - [9] B. Link and R. I. Epstein, *Astrophys. J.* **457**, 844 (1996); M. B. Larson and B. Link, *Mon. Not. R. Astron. Soc.* **317**, 843 (2000).
 - [10] M. Ruderman, T. Zhu, and K. Cheng, *Astrophys. J.* **492**, 267 (1998).
 - [11] B. Carter, D. Langlois, and D. M. Sedrakian, *Astron. Astrophys.* **361**, 795 (2000).
 - [12] K. S. Cheng, D. Pines, M. A. Alpar, and J. Shaham, *Astrophys. J.* **330**, 835 (1988).
 - [13] A. Sedrakian and J. M. Cordes, *Mon. Not. R. Astron. Soc.* **307**, 365 (1999).
 - [14] M. A. Alpar, H. F. Chau, K. S. Cheng, and D. Pines, *Astrophys. J.* **409**, 345 (1993).
 - [15] P. B. Jones, *Mon. Not. R. Astron. Soc.* **243**, 257 (1990); *Astrophys. J.* **373**, 208 (1991).
 - [16] L. M. Franco, B. Link, and R. I. Epstein, *Astrophys. J.* **543**, 987 (2000).
 - [17] P. B. Jones, *Mon. Not. R. Astron. Soc.* **335**, 733 (2002).
 - [18] T. Nozawa, N. Stergioulas, E. Gourgoulhon, and Y. Eriguchi, *Astron. Astrophys., Suppl. Ser.* **132**, 431 (1998).
 - [19] W. Schönauer and R. Weiß, *J. Comput. Appl. Math.* **27**, 279 (1989); M. Schauder, R. Weiß and W. Schönauer, *The CADSOL Program Package*, Universität Karlsruhe, Interner Bericht Nr. 46/92 (1992).
 - [20] B. Kleihaus, J. Kunz, and F. Navarro-Lerida, *Phys. Rev. Lett.* **90**, 171101 (2003).